

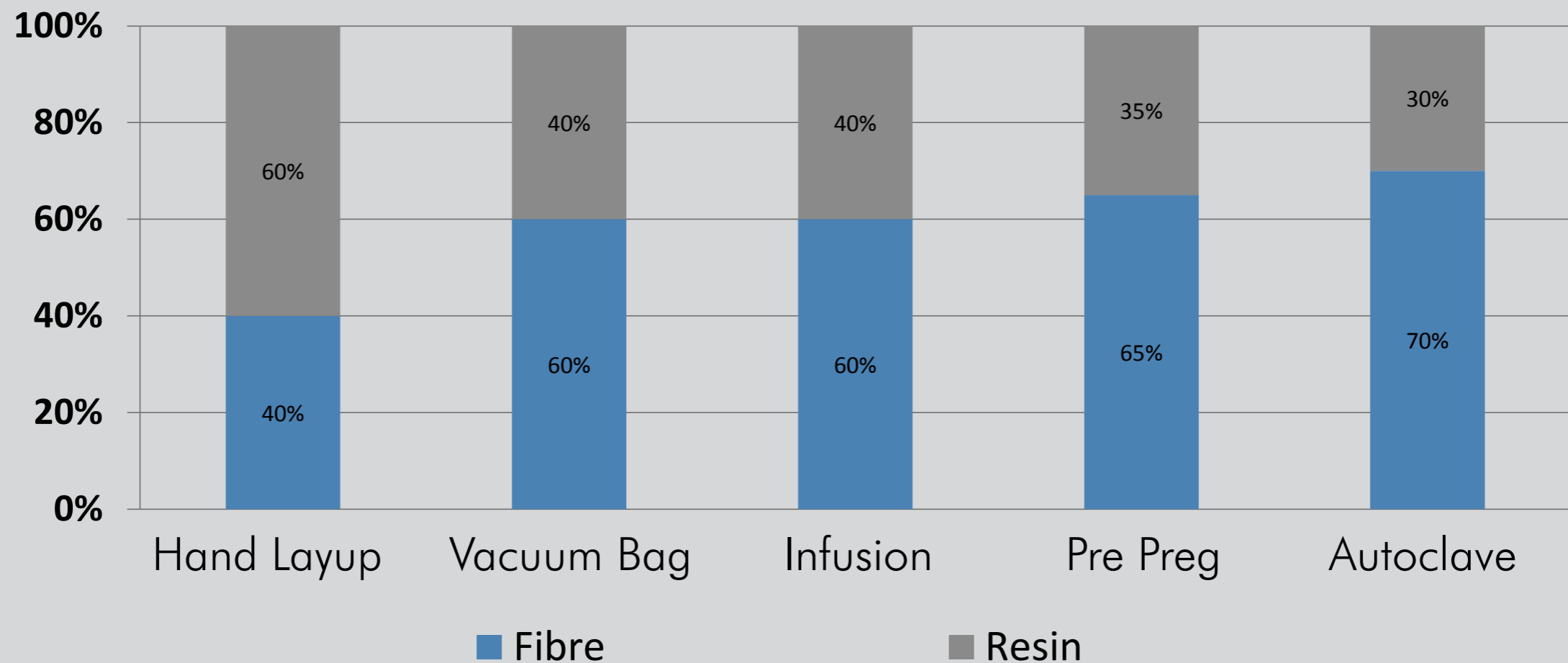
COMPOSITE MATERIALS

MECHANICS

# COMPOSITE MATERIALS MECHANICS

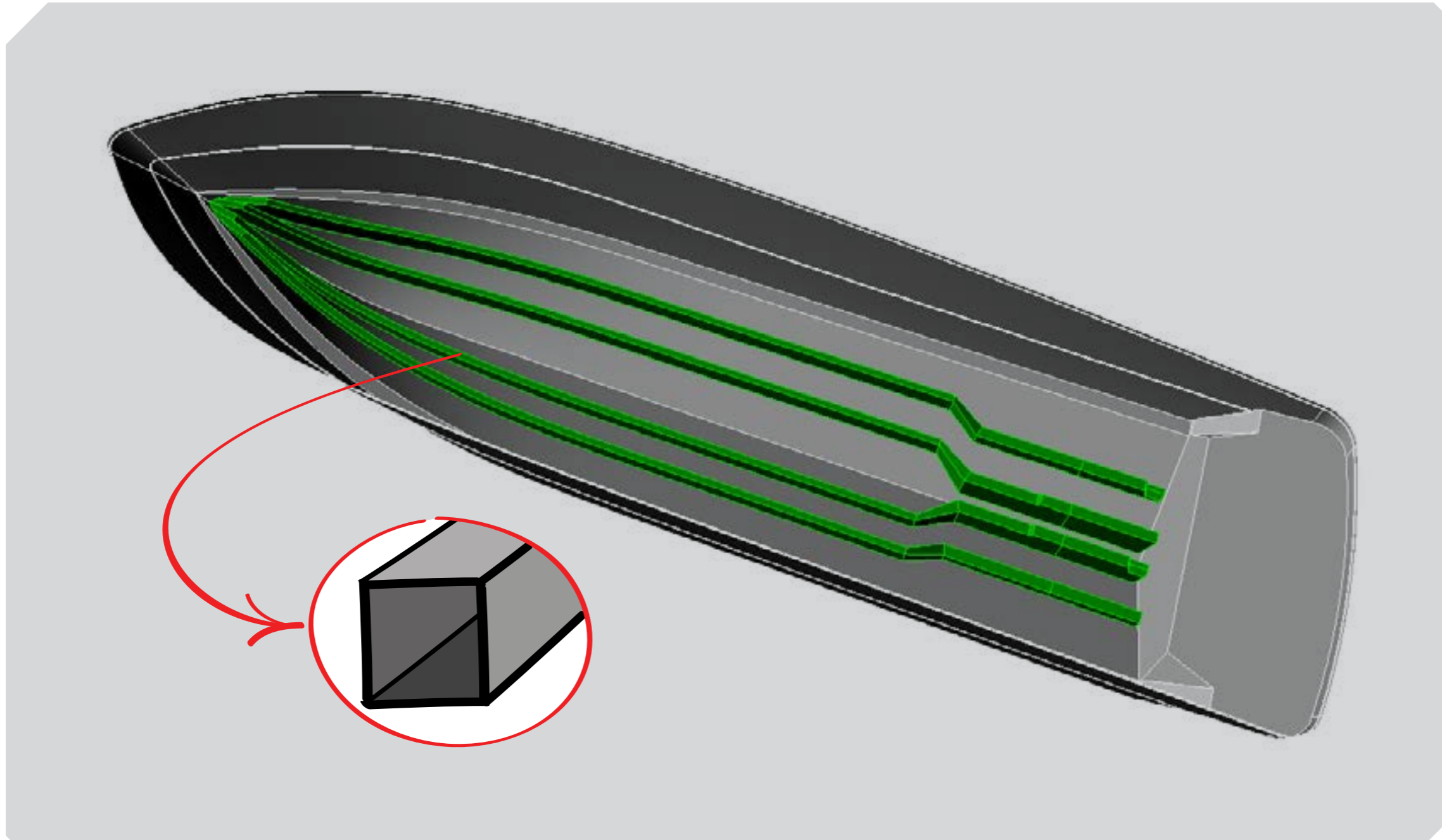
## FIBER FRACTION

Fibre Content x Manufacturing Process



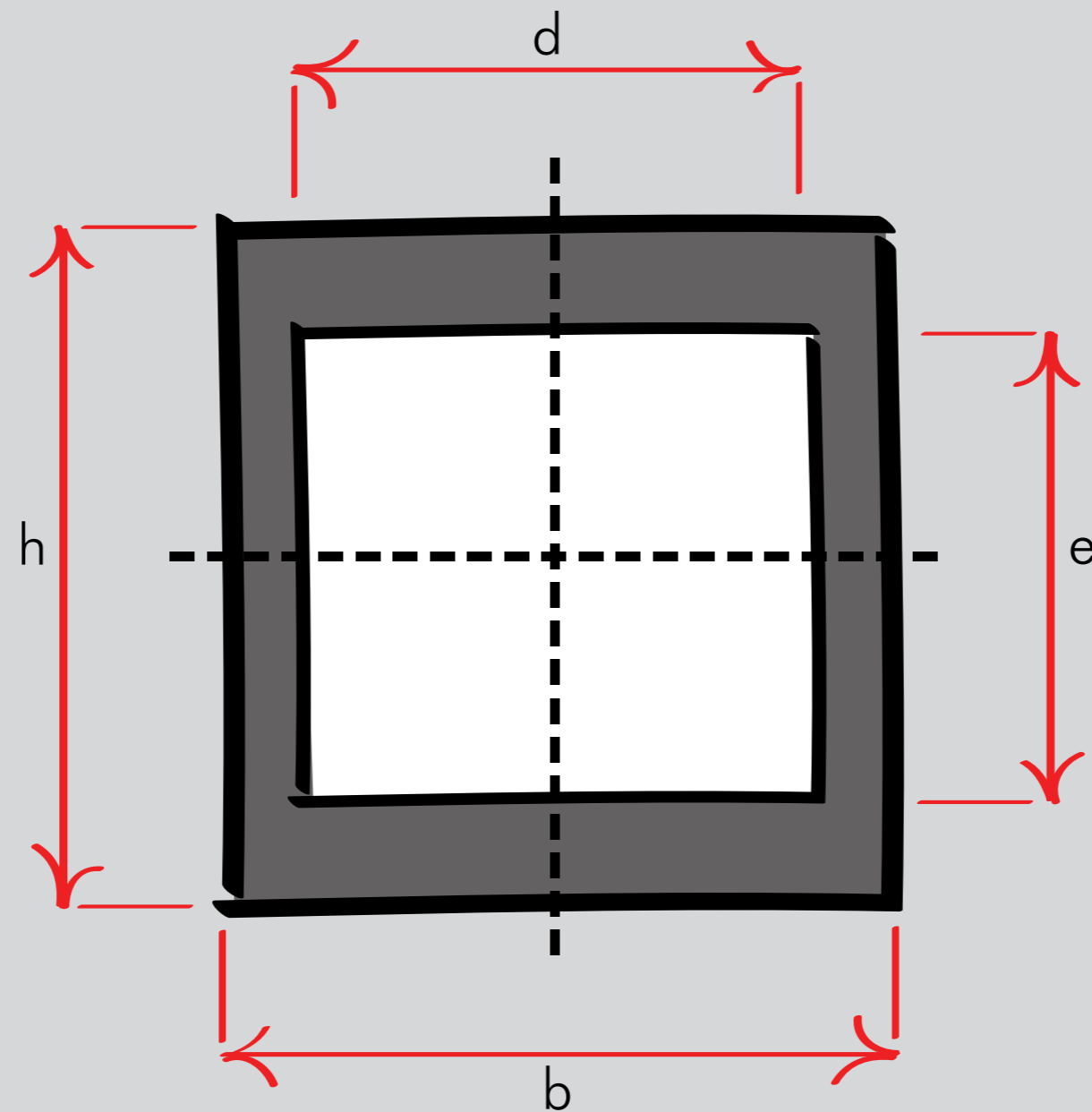
# COMPOSITE MATERIALS MECHANICS

## EXAMPLE



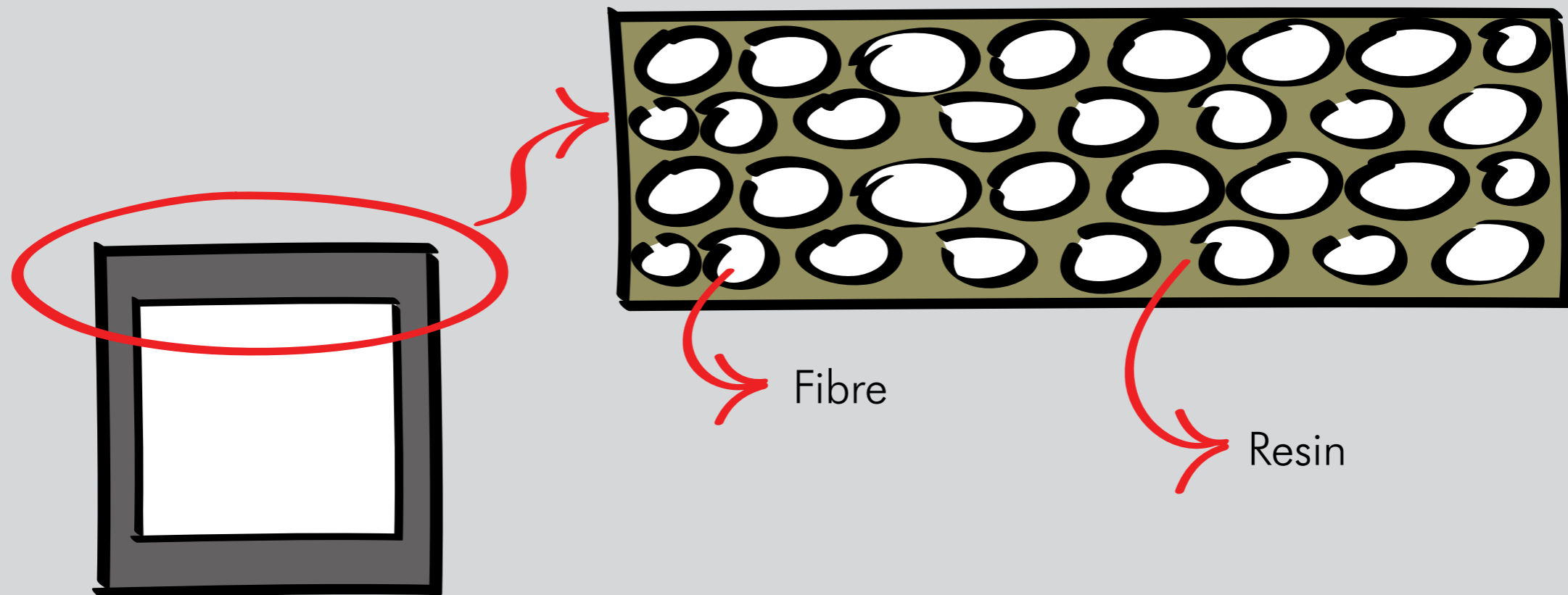
# COMPOSITE MATERIALS MECHANICS

## EXAMPLE



# COMPOSITE MATERIALS MECHANICS

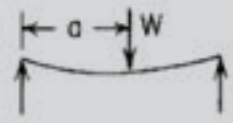
## EXAMPLE



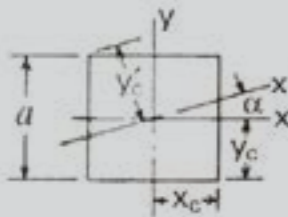
# COMPOSITE MATERIALS MECHANICS

## BASIC PRINCIPLES

**TABLE 8.1 Shear, moment, slope, and deflection formulas for elastic straight beams (Continued)**

End restraints, reference no.	Boundary values	Selected maximum values of moments and deformations
1e. Left end simply supported, right end simply supported 	$R_A = \frac{W}{l}(l-a) \quad M_A = 0$ $\theta_A = \frac{-Wa}{6EI}(2l-a)(l-a) \quad y_A = 0$ $R_B = \frac{Wa}{l} \quad M_B = 0$ $\theta_B = \frac{Wa}{6EI}(l^2 - a^2) \quad y_B = 0$	$\text{Max } M = R_A a \text{ at } x = a; \text{ max possible value} = \frac{Wl}{4} \text{ when } a = \frac{l}{2}$ $\text{Max } y = \frac{-Wa}{3EI} \left(\frac{l^2 - a^2}{3}\right)^{3/2} \text{ at } x = l - \left(\frac{l^2 - a^2}{3}\right)^{1/2} \text{ when } a < \frac{l}{2}; \text{ max possible value} = \frac{-Wl^3}{48EI} \text{ at } x = \frac{l}{2} \text{ when } a = \frac{l}{2}$ $\text{Max } \theta = \theta_A \text{ when } a < \frac{l}{2}; \text{ max possible value} = -0.0642 \frac{Wl^2}{EI} \text{ when } a = 0.423l$

Notation:  $A$  = area (length<sup>2</sup>);  $y$  = distance to extreme fiber (length);  $I$  = moment of inertia (length<sup>4</sup>);  $r$  = radius of gyration (length);  $Z$  = plastic section modulus (length<sup>3</sup>); SF = shape factor. See Sec. 8.15 for applications of  $Z$  and SF.

Form of Section	Area and Distances from Centroid to Extremities	Moments and Products of Inertia and Radii of Gyration about Central Axes	Plastic Section Moduli, Shape Factors, and Locations of Plastic Neutral Axes
1. Square 	$A = a^2$ $y_c = x_c = \frac{a}{2}$ $y'_c = 0.707a \cos\left(\frac{\pi}{4} - \alpha\right)$	$I_x = I_y = I'_x = \frac{1}{12}a^4$ $r_x = r_y = r'_x = 0.2887a$	$Z_x = Z_y = 0.25a^3$ $SF_x = SF_y = 1.5$

# COMPOSITE MATERIALS MECHANICS

## HOW MUCH REINFORCEMENT?

### Weight Fraction

Used in manufacture. May refer to fibre or resin - 'GRP' manufacturers will specify a glass content of (e.g.) 25 wt%; a prepreg supplier might give a resin content of 35 wt%.

### Volume Fraction

Used in design to calculate composite properties.  
Almost always refers to fibre content.

# COMPOSITE MATERIALS MECHANICS

## CONVERSION

### Weight Fraction - Volume Fraction

For the Special Case of a Two-Component Composite  
(eg Fibre and Matrix)

$$V_f = \frac{W_f / \rho_f}{W_f / \rho_f + (1 - W_f) / \rho_m}$$

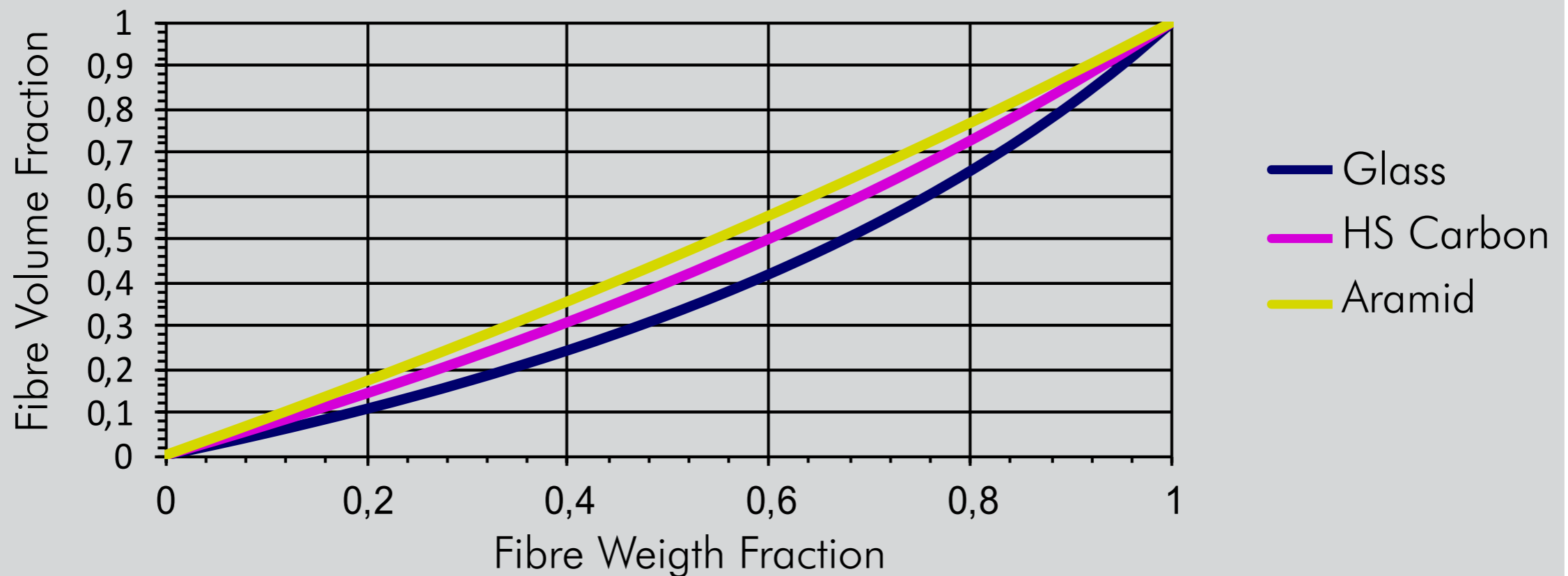
$$W_f = \frac{\rho_f V_f}{\rho_f V_f + \rho_m (1 - V_f)}$$



# COMPOSITE MATERIALS MECHANICS

## CONVERSION

Weight Fraction - Volume Fraction  
(Epoxy Resin Matrix)

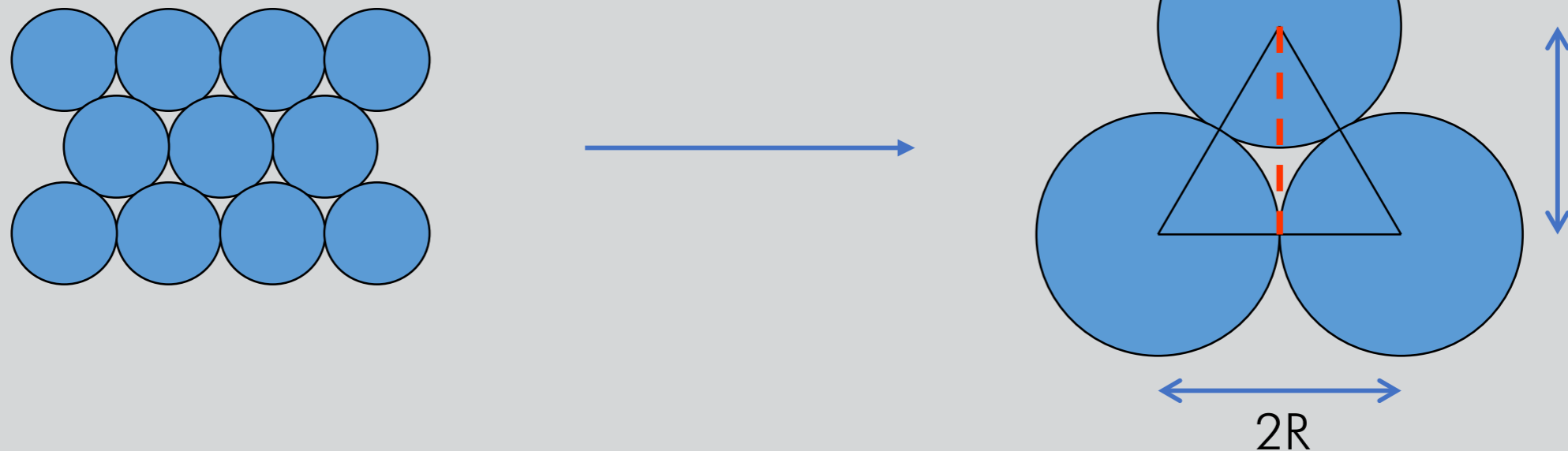


# COMPOSITE MATERIALS MECHANICS

## MAXIMUM FRACTION

### Maximum Fibre Volume Fraction

A composite cannot contain 100% fibre. Maximum volume fraction could be achieved only if unidirectional fibres are hexagonally 'close packed' - ie all fibres are touching.



The triangular unit cell has area  $\sqrt{3}R^2$ .

The unit cell contains an area of fibre (three 60 segments) equal to  $\pi R^2/2$

# COMPOSITE MATERIALS MECHANICS

## MAXIMUM FRACTION

### Maximum Fibre Volume Fraction

In a unidirectional fibre composite, the fibre area fraction is the same as the fibre volume fraction, so:

$$V_f^{\max} = \frac{\pi R^2 / 2}{\sqrt{3} R^2} = \frac{\pi}{2\sqrt{3}} = 0.908 \approx 91\%$$

# COMPOSITE MATERIALS MECHANICS

## MAXIMUM FRACTION

### Maximum Fibre Volume Fraction

Theoretically, a unidirectional fibre composite could have  $V_f \approx 91\%$ .  
In practice, fibres cannot be perfectly aligned.

Maximum volume fraction depends both on the fibre form and method of manufacture - for a unidirectional fibre composite:  $V_f \approx 70-72\%$ .

# COMPOSITE MATERIALS MECHANICS

## MAXIMUM FRACTION

### Maximum Fibre Volume Fraction

For other forms of reinforcement, maximum volume fraction also depends on the detailed arrangement of the fibres.

The following values are typical:

Unidirectional	0.70 – 0.72
stitched 'non-crimp'	0.60 – 0.65
woven fabric random	0.40 - 0.55
chopped strand mat	0.15 - 0.25

# COMPOSITE MATERIALS MECHANICS

## HOW MUCH FIBRE?

Commercial reinforcements are characterised by their areal weight ( $A_w$ ). This is simply the weight (usually given in g) of 1 m<sup>2</sup> of the reinforcement.  $A_w$  depends on many factors - fibre density, tow or bundle size, weave style, etc.

$A_w$  may range from 50 g/m<sup>2</sup> or less (for lightweight surfacing tissues), up to more than 2000 g/m<sup>2</sup> for some heavyweight non-crimp fabrics.

# COMPOSITE MATERIALS MECHANICS

## LAMINATE THICKNESS

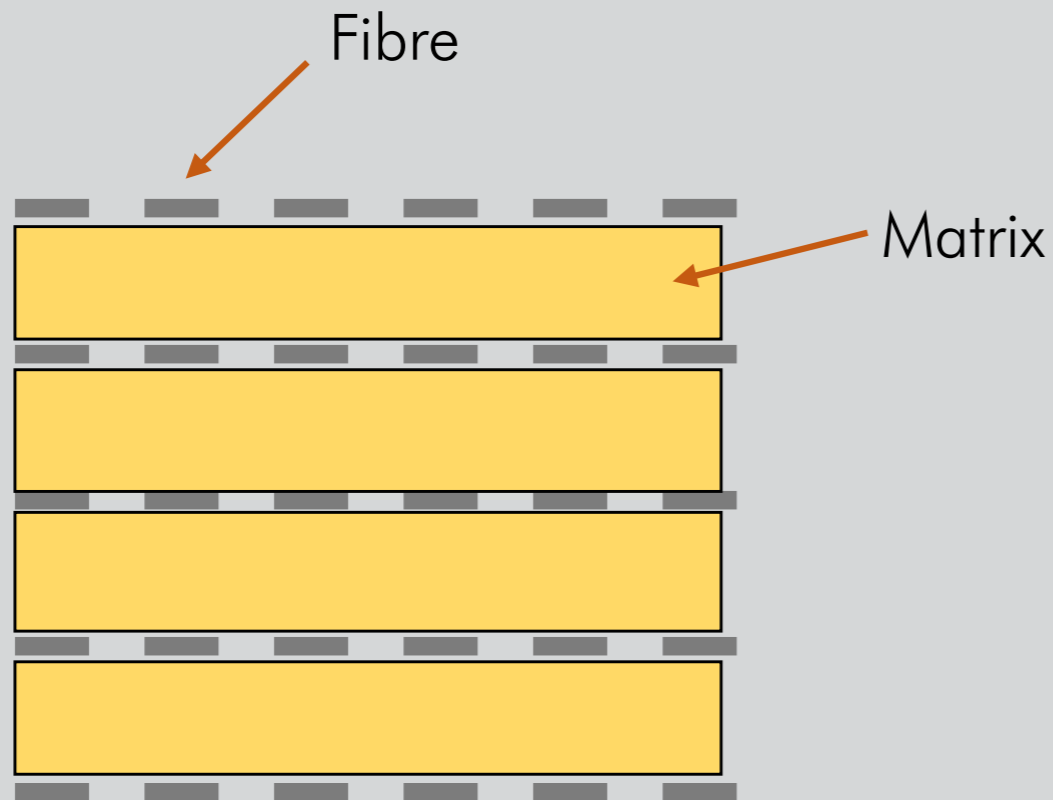
The thickness of a composite laminate depends on the amount of reinforcement and the relative amount of resin which has been included.

For a given quantity of reinforcement, a laminate with a high fibre volume fraction will be thinner than one with a lower fibre volume fraction, since it will contain less resin.

# COMPOSITE MATERIALS MECHANICS

## LAMINATE THICKNESS

Two Laminates, Both Containing 5 Plies of Reinforcement



high matrix content = low fibre content  
= thick laminate



low matrix content = high fibre content  
= thin laminate



# COMPOSITE MATERIALS MECHANICS

## LAMINATE THICKNESS

Fibre Volume Fraction is Thus Inversely Proportional to Laminate Thickness.

If the fibre content and laminate thickness are defined, we can calculate the fibre volume fraction

$$V_f = \frac{nA_w}{\rho_f d}$$

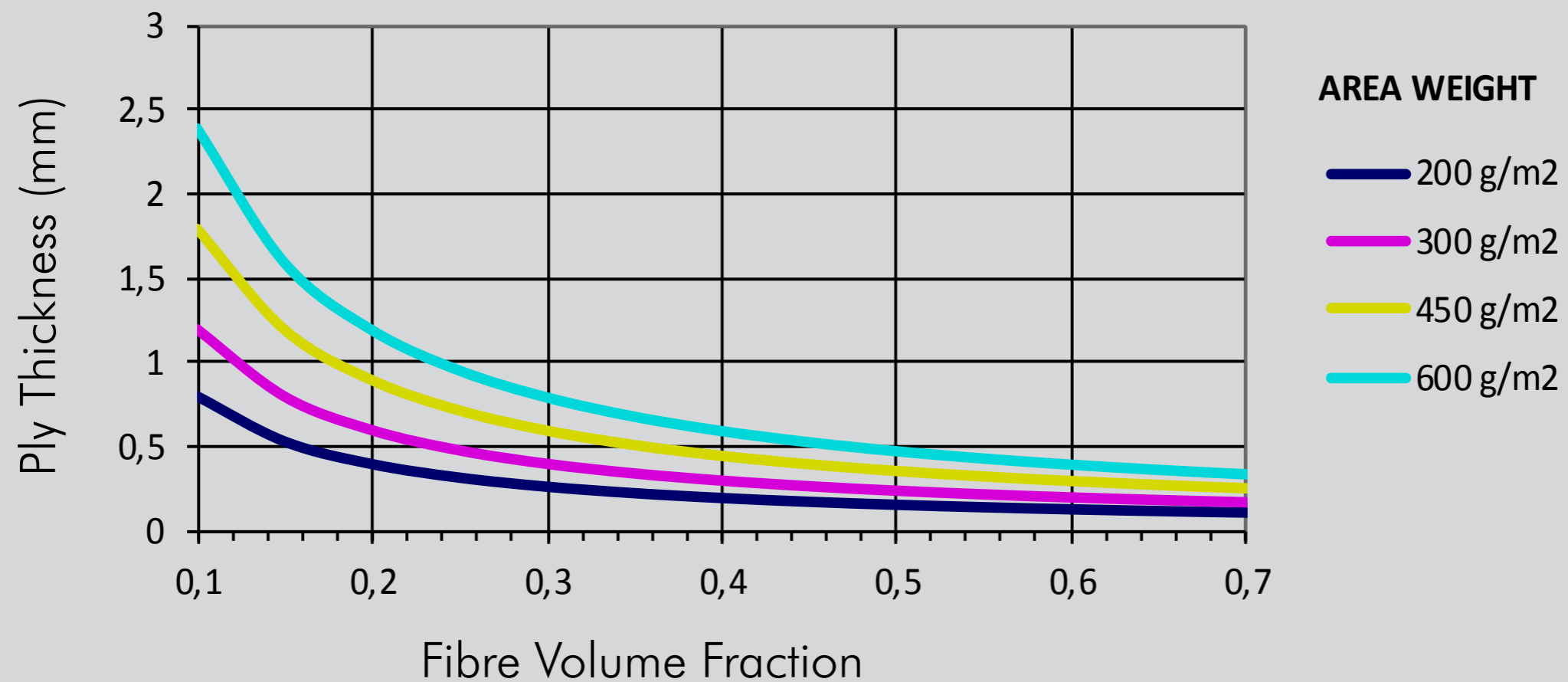
If the fibre content and volume fraction are defined, we can calculate the laminate thickness

$$d = \frac{nA_w}{\rho_f V_f}$$

# COMPOSITE MATERIALS MECHANICS

## LAMINATE THICKNESS

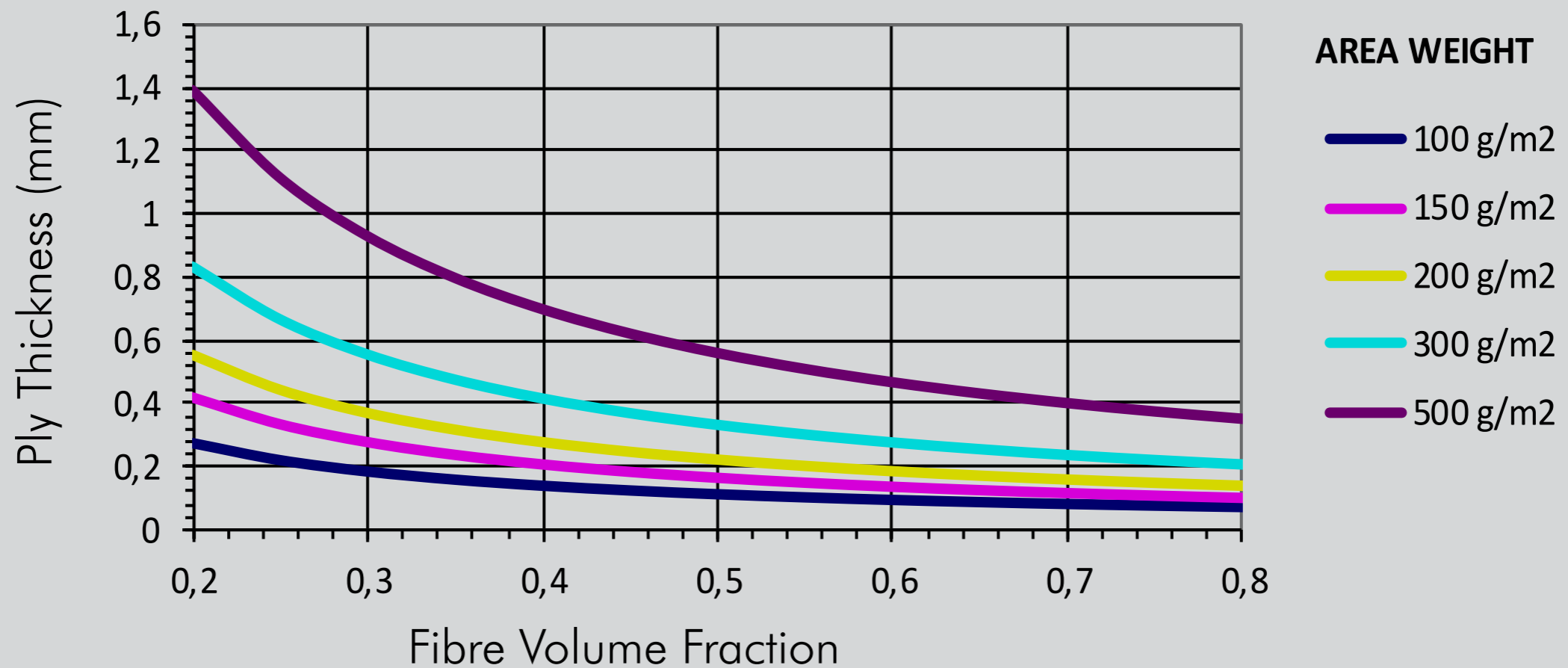
Ply Thickness x Fibre Volume Fraction  
(Glass)



# COMPOSITE MATERIALS MECHANICS

## LAMINATE THICKNESS

Ply Thickness x Fibre Volume Fraction  
(HS Carbon)



# COMPOSITE MATERIALS MECHANICS

## MIXTURE RULES

### Rules of Mixture for Elastic Properties

'Rules of Mixtures' are mathematical expressions which give some property of the composite in terms of the properties, quantity and arrangement of its constituents.

They may be based on a number of simplifying assumptions, and their use in design should be tempered with extreme caution!

# COMPOSITE MATERIALS MECHANICS

## DENSITY

For the Special Case of a Fibre-Reinforced Matrix

$$\rho = V_f \rho_f + V_m \rho_m$$

$$\rho = V_f \rho_f + (1 - V_f) \rho_m$$

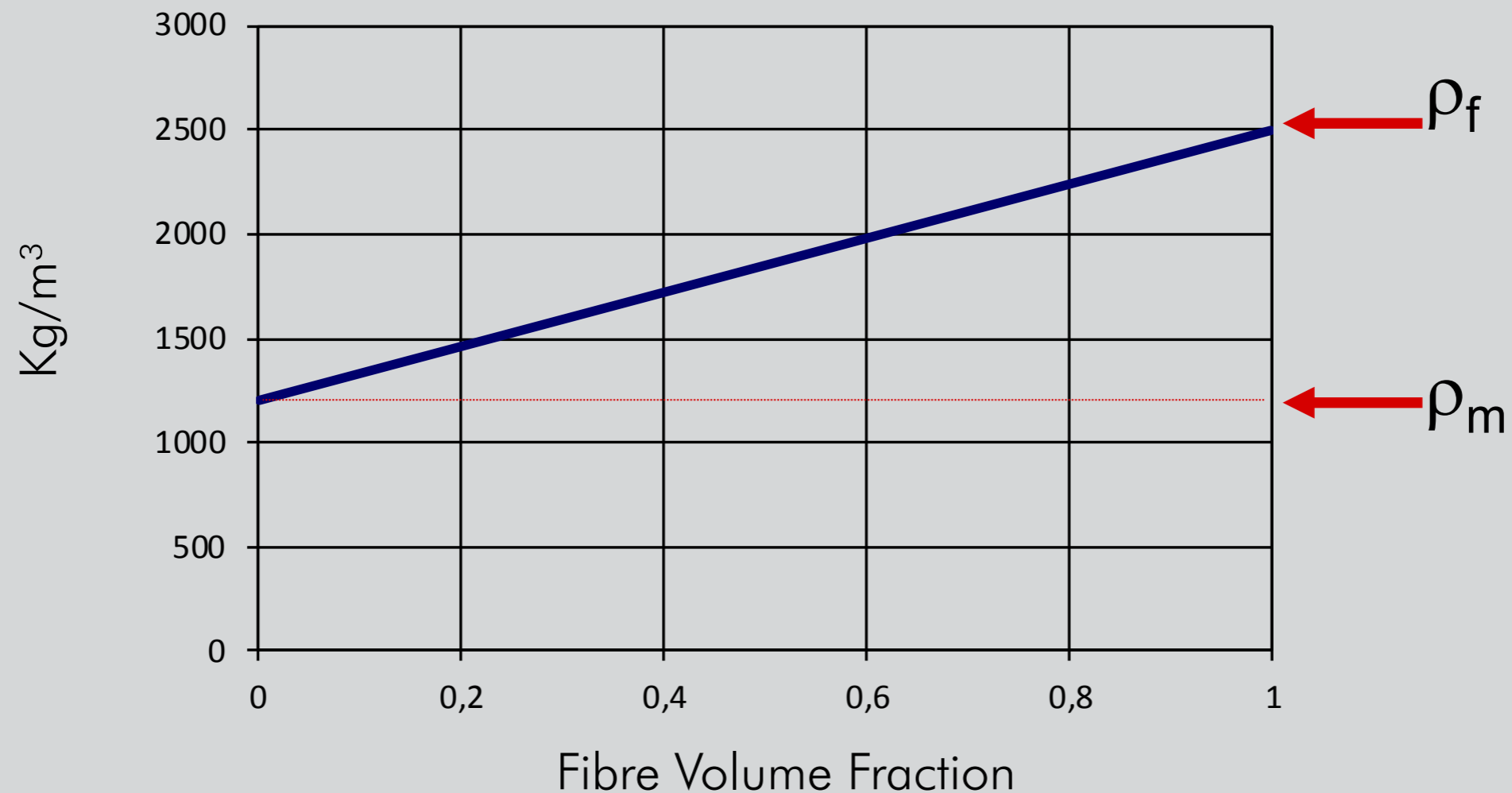
$$\rho = V_f (\rho_f - \rho_m) + \rho_m$$

Since  $V_f + V_m = 1$

# COMPOSITE MATERIALS MECHANICS

## DENSITY

### Rule of Mixtures Density for Glass/Epoxy Composites



# COMPOSITE MATERIALS MECHANICS

## TENSILE MODULUS

### Generalised Rule of Mixtures for Tensile Modulus

$$E = \eta_L \eta_o E_f V_f + E_m (1 - V_f)$$

$\eta_L$  is a length correction factor. Typically,  $\eta_L = 1$  for fibres longer than about 10 mm.

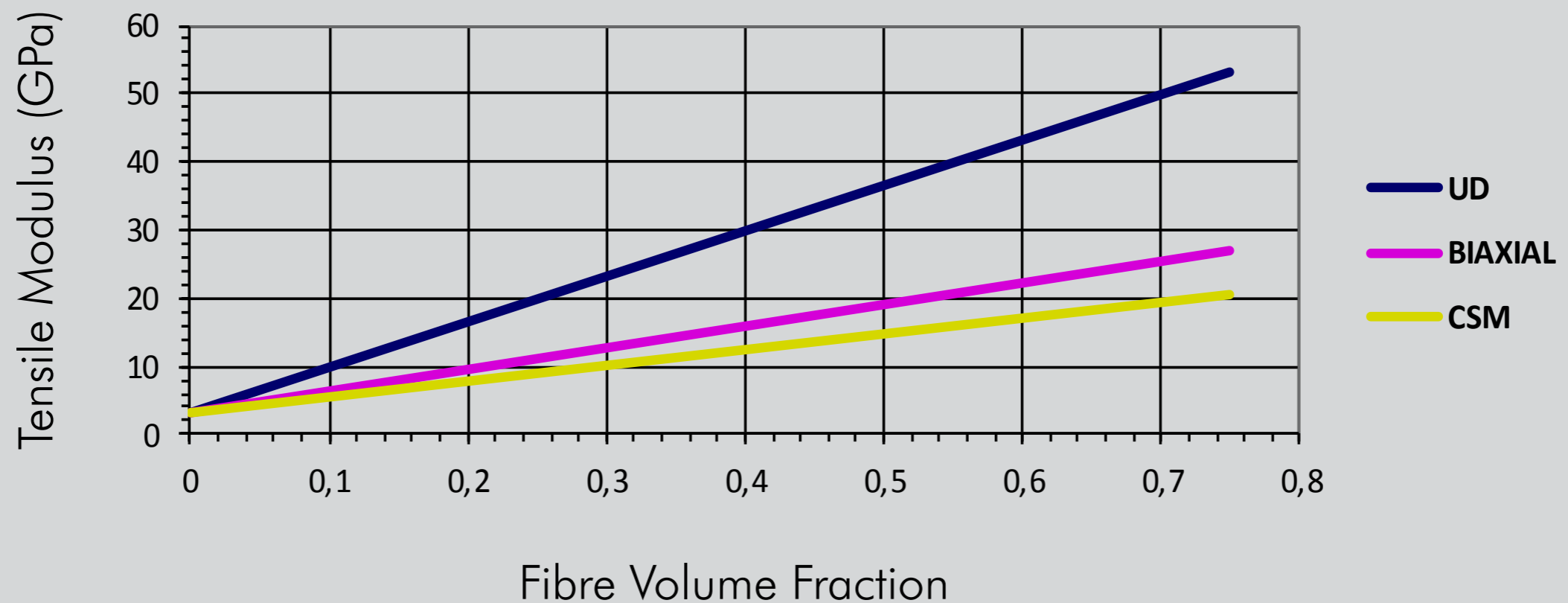
$\eta_o$  corrects for non-unidirectional reinforcement:

	$\eta_o$
Unidirectional	1.0
Biaxial	0.5
Biaxial at $\pm 45^\circ$	0.25
Random (in-plane)	0.375
Random (3D)	0.2

# COMPOSITE MATERIALS MECHANICS

## TENSILE MODULUS

### Rule of Mixtures Tensile Modulus (Glass Fibre / Polyester)

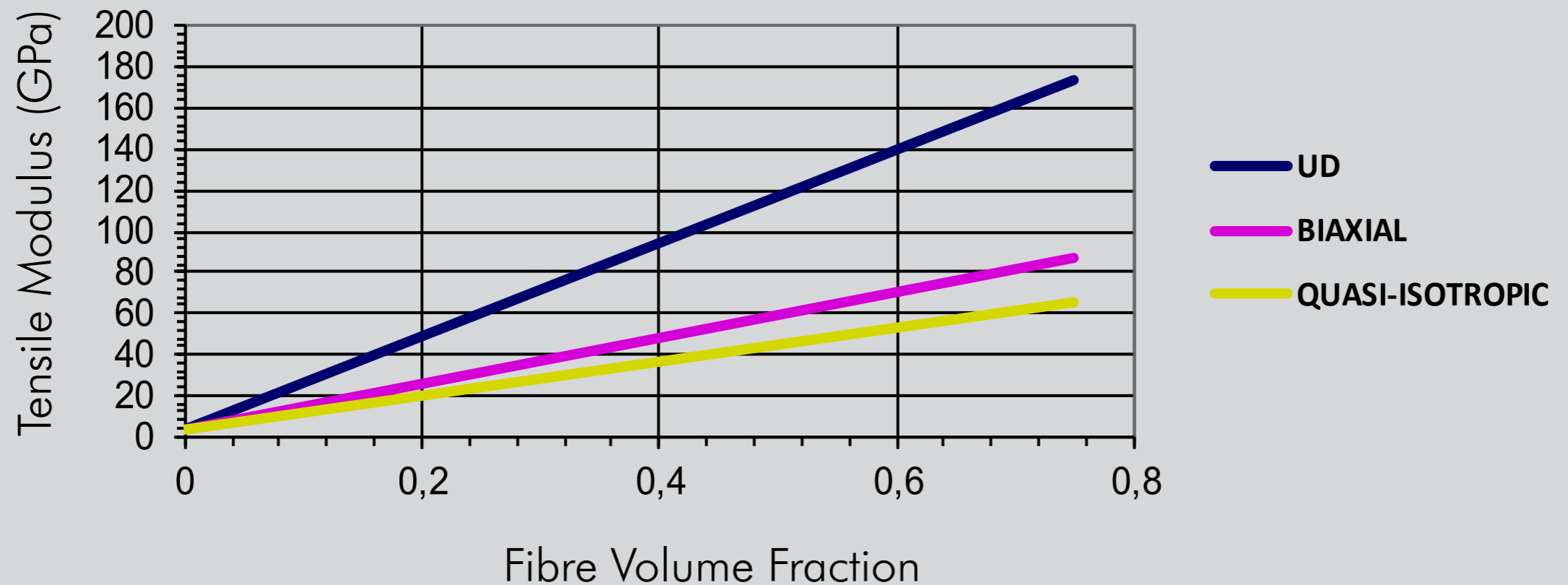




# COMPOSITE MATERIALS MECHANICS

## TENSILE MODULUS

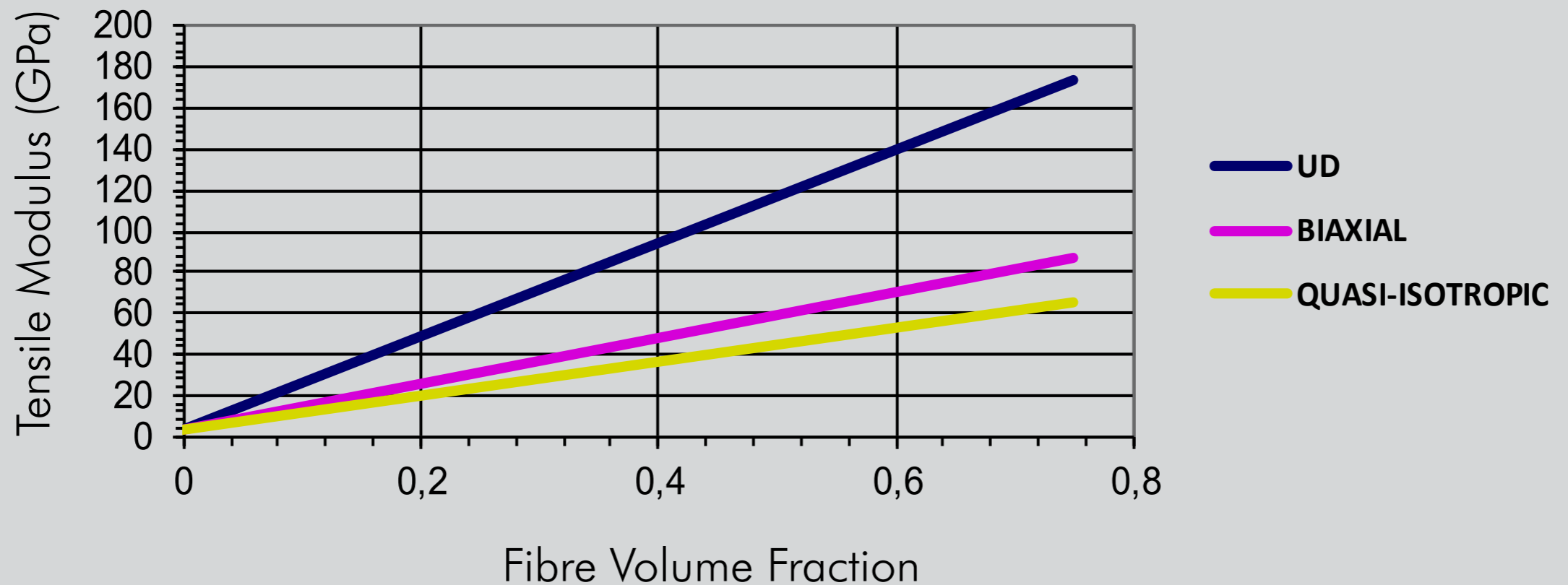
Rule of Mixtures Tensile Modulus  
(Carbon Fibre)



# COMPOSITE MATERIALS MECHANICS

## TENSILE MODULUS

Rule of Mixtures Tensile Modulus  
(Carbon Fibre)



# COMPOSITE MATERIALS MECHANICS

Obrigado!